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Assignment 6: Optics II      Due: Wed. March 1. 6:00PM

PART 1 Provide the concise note on Mirrors and Lenses and Image Formation.

This hand written note should fit on 3 pages (this page and its back and the back of the page on the PART II other page note should include

a) fundamental equations (without their derivation) for refractive surface imaging, mirror equation, lens maker equation and thin lens equation.

b) detailed description of an eye.

c) detailed description of the simple magnifier, compound microscope and refracting telescope.

## PART II PROBLEMS

1. An antelope is at a distance of 20.0 m from a converging lens of focal length 30.0 cm. The lens forms an image of the animal. If the antelope runs away from the lens at a speed of 5.00 m/s, how fast does the image move? Does the image move toward or away from the lens?

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \text{by implicit differentiation} \Rightarrow -\frac{1}{p^2} \frac{dp}{dt} - \frac{1}{q^2} \frac{dq}{dt} = 0 \text{ and so we have } -\frac{q^2}{p^2} v_p = v_q$$

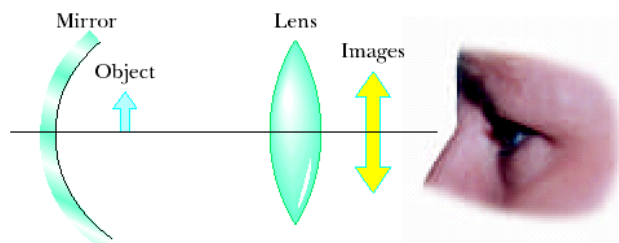
since  $q = 0.30457 \text{ m}$   $v_q = -0.00115 \text{ m/s} = -1.16 \text{ mm/s}$ .

If antelope runs away from the lens the image of it runs towards the lens.

BTW It is OK to solve this problem using two positions (found by using original  $p$  and  $v_p$ ).

However, one must use very small time increments. (1 second is way too much!!!) and consequently keep answers unrounded which is tedious and exhausting.

2. An observer to the right of the mirror-lens combination shown on diagram sees two real images that are the same size and in the same location. One image is upright and the other is inverted. Both images are 1.50 times larger than the object. The lens has a focal length of 10.0 cm. The lens and mirror are separated by 40.0 cm. Determine the focal length of the mirror. Do not assume that the figure is drawn to scale.



The inverted real image is formed by the lens operating on light directly from the object, on light that has not reflected from the mirror.

For this we have  $M = -1.50 = -\frac{q}{p}$   $q = 1.50p$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{p} + \frac{1}{1.50p} = \frac{1}{10 \text{ cm}} = \frac{2.50}{1.50p}$$

$$p = 10 \text{ cm} \left( \frac{2.5}{1.5} \right) = 16.7 \text{ cm}$$

Then the object is distant from the mirror by  $40.0 \text{ cm} - 16.7 \text{ cm} = 23.3 \text{ cm}$ .

The second image seen by the person is formed by light that first reflects from the mirror and then goes through the lens. For it to be in the same position as the inverted image, the lens must be receiving light from an image formed by the mirror at the same location as the physical object. The formation of this image is described by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{23.3 \text{ cm}} + \frac{1}{23.3 \text{ cm}} = \frac{1}{f}$$

$$f = \boxed{11.7 \text{ cm}}.$$

3. Two lenses made of kinds of glass having different refractive indices  $n_1$  and  $n_2$  are cemented together to form what is called an *optical doublet*. Optical doublets are often used to correct chromatic aberrations in optical devices. The first lens of a doublet has one flat side and one concave side of radius of curvature  $R$ . The second lens has two convex sides of radius of curvature  $R$ . Show that the doublet can be modeled as a single thin lens with a focal length described by

$$\frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}$$

The first lens has focal length described by

$$\frac{1}{f_1} = (n_1 - 1) \left( \frac{1}{R_{11}} - \frac{1}{R_{12}} \right) = (n_1 - 1) \left( \frac{1}{\infty} - \frac{1}{R} \right) = -\frac{n_1 - 1}{R}.$$

For the second lens

$$\frac{1}{f_2} = (n_2 - 1) \left( \frac{1}{R_{21}} - \frac{1}{R_{22}} \right) = (n_2 - 1) \left( \frac{1}{+R} - \frac{1}{-R} \right) = +\frac{2(n_2 - 1)}{R}.$$

Let an object be placed at any distance  $p_1$  large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \quad \text{and thus:} \quad \frac{1}{q_1} = \frac{-n_1 + 1}{R} - \frac{1}{p_1}.$$

This virtual ( $q_1 < 0$ ) image is a real object for the second lens at distance  $p_2 = -q_1$ . For the second lens

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \quad \text{so that:} \quad \frac{1}{q_2} = \frac{2n_2 - 2}{R} - \frac{1}{p_2} = \frac{2n_2 - 2}{R} + \frac{1}{q_1} = \frac{2n_2 - 2}{R} + \frac{-n_1 + 1}{R} - \frac{1}{p_1} = \frac{2n_2 - n_1 - 1}{R} - \frac{1}{p_1}.$$

Then  $\frac{1}{p_1} + \frac{1}{q_2} = \frac{2n_2 - n_1 - 1}{R}$  so the doublet behaves like a single lens with  $\frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}$ .